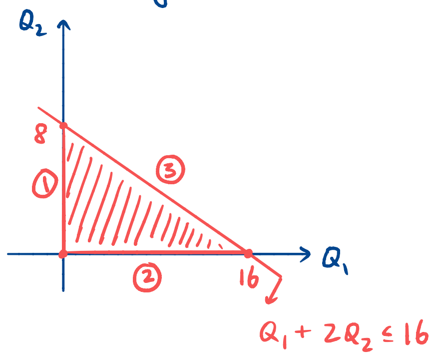
Ex. 2

Feasible region:



Critical points:

$$\nabla\pi(Q_1, Q_2) = \begin{bmatrix} 5 - \frac{2}{3}Q_1 \\ 4 - Q_2 \end{bmatrix}$$

Solve $\nabla\pi = 0$:

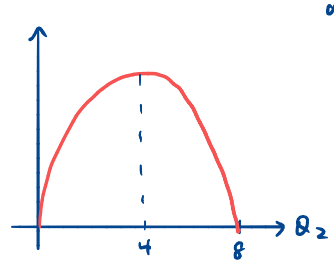
CPs: $\left(\frac{15}{2}, 4\right)$

$\left(\frac{15}{2}, 4\right)$ is <u>in</u> the feasible region. $\pi\left(\frac{15}{2}, 4\right) = 26.75$

Line segment ①: $Q_1 = 0$

$\Rightarrow \pi(Q_1, Q_2) = \pi(0, Q_2) = 4Q_2 - \frac{1}{2}Q_2^2$

$\frac{\partial\pi(0, Q_2)}{\partial Q_2} = 4 - Q_2 \Rightarrow$ parabola vertex at $Q_2 = 4$

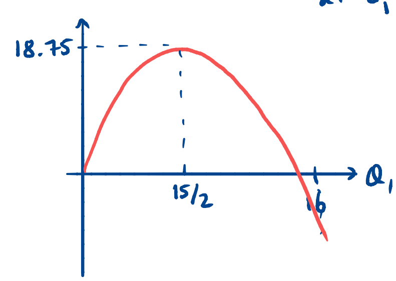


$\Rightarrow \pi(0, 4) = 8$ is the maximum value on line segment ①

Line segment ②: $Q_2 = 0$

$\Rightarrow \pi(Q_1, Q_2) = \pi(Q_1, 0) = 5Q_1 - \frac{1}{3}Q_1^2$

$\frac{\partial\pi(Q_1, 0)}{\partial Q_1} = 5 - \frac{2}{3}Q_1 \Rightarrow$ parabola vertex at $Q_1 = \frac{15}{2}$



$\Rightarrow \pi\left(\frac{15}{2}, 0\right) = 18.75$ is the maximum value on line segment ②

Line segment ③: $Q_1 + 2Q_2 = 16$

$\Rightarrow \pi(Q_1, Q_2) = \pi(16 - 2Q_2, Q_2)$

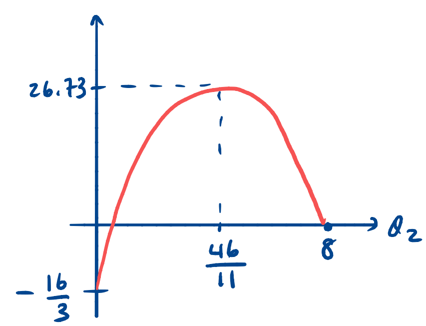
$= 5(16 - 2Q_2) - \frac{1}{3}(16 - 2Q_2)^2 + 4Q_2 - \frac{1}{2}Q_2^2$

$= 80 - 10Q_2 - \frac{4}{3}(64 - 16Q_2 + Q_2^2) + 4Q_2 - \frac{1}{2}Q_2^2$

$= -\frac{16}{3} + \frac{46}{3}Q_2 - \frac{11}{6}Q_2^2$

$\frac{\partial\pi(16 - 2Q_2, Q_2)}{\partial Q_2} = \frac{46}{3} - \frac{11}{3}Q_2 \Rightarrow$ parabola vertex at $Q_2 = \frac{46}{11}$

$\left(16 - 2Q_2 = \frac{84}{11}\right)$



$\Rightarrow \pi\left(\frac{84}{11}, \frac{46}{11}\right) \approx 26.73$ is the maximum value on line segment ③

Ex. 2 cont.

$\Rightarrow \pi\left(\frac{15}{2}, 4\right) = 26.75$ is an absolute maximum.

$\Rightarrow$ The company should manufacture $\frac{15}{2}$ units of product 1 and 4 units of product 2, for a profit of 26.75.